Exponential Functions

Which of these are exponential functions of x? x^3, 3^x, 3^{-x}, $x^{1/3}$, $\left(\frac{1}{3}\right)^x$

Simplify the following $5^{10} \times 5^2$, $7^3 \div 7^5$, $(3^2)^5$, $4^{10} \times 6^{10}$

Simplify $\left(\exp(3+2)\right)^2 \div \left(\exp(5-2)\right)^3$

Give the values of x which satisfy $\dfrac{\left(5^{15x} \times 5^{-24}\right)^{1/3}}{5^{x^2}} = 0.04$

Exponential Functions

Exponential $f(x) = a^{x}$, $a > 0$ constant

exponent

Power functions $f(x) = x^{a}$

base

Which of these are exponential functions of x?

✗ x^{3} ← base power function

3^{x} ← exponent ✓

$\left(\frac{1}{3}\right)^{x}$ ← exponent ✓

$x^{1/3}$ ← base power function ✗

$3^{-x} = \frac{1}{3^{x}} = \frac{1^{x}}{3^{x}} = \left(\frac{1}{3}\right)^{x}$ ← exponent ✓

↑ base > 0

$a^{-b} = \frac{1}{a^{b}}$

$\left(\frac{a}{b}\right)^{c} = \frac{a^{c}}{b^{c}}$

Exponential Functions

Simplify the following

$$a^x a^y = a^{x+y}$$

$$5^{10} \times 5^2 = 5^{10+2} = \underline{\underline{5^{12}}}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$7^3 \div 7^5 = 7^{3-5} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$$

$$a^{-x} = \frac{1}{a^x}$$

$$(3^2)^5 = 3^{2\times 5} = \underline{\underline{3^{10}}}$$

$$(a^x)^y = a^{xy}$$

$$4^{10} \times 6^{10} = (4\times 6)^{10} = \underline{\underline{24^{10}}}$$

$$a^x b^x = (ab)^x$$

Exponential Functions

Simplify $\left(\exp(3+2)\right)^2 \div \left(\exp(5-2)\right)^3 = \dfrac{(e^5)^2}{(e^3)^3} = \dfrac{e^{5\times 2}}{e^{3\times 3}} = \dfrac{e^{10}}{e^9} = e^{10-9}$

$= e^1$

$= e$

$\exp(3+2) = \exp(5) = e^5$

$\exp(5-2) = \exp(3) = e^3$

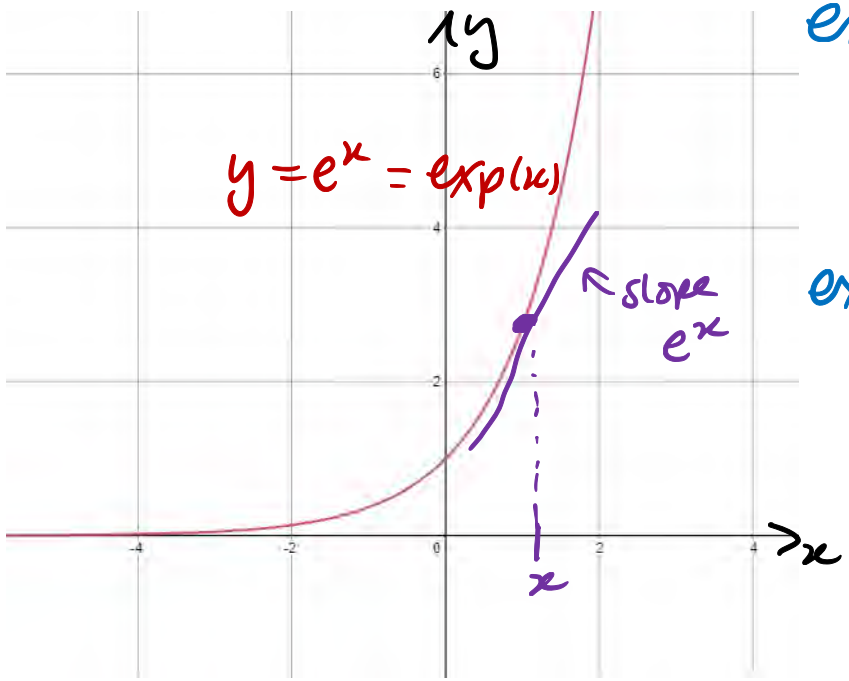
$a^x a^y = a^{x+y}$

$\dfrac{a^x}{a^y} = a^{x-y}$

$a^{-x} = \dfrac{1}{a^x}$

$(a^x)^y = a^{xy}$

$a^x b^x = (ab)^x$



exp

natural exponential function

$\exp(x) = e^x$

Euler's number

$e \approx 2.71$

Exponential Functions

Give the values of x which satisfy $\dfrac{\left(5^{15x} \times 5^{-24}\right)^{1/3}}{5^{x^2}} = 0.04$ — (1)

$$5^{15x} \times 5^{-24} = 5^{15x+(-24)} = 5^{15x-24}$$

$$\left(5^{15x-24}\right)^{\frac{1}{3}} = 5^{(15x-24)\times\frac{1}{3}}$$

$$= 5^{5x-8}$$

LHS: $\dfrac{5^{5x-8}}{5^{x^2}} = 5^{5x-8-x^2}$, RHS $0.04 = \dfrac{4}{100} = \dfrac{1}{25} = \dfrac{1}{5^2} = 5^{-2}$

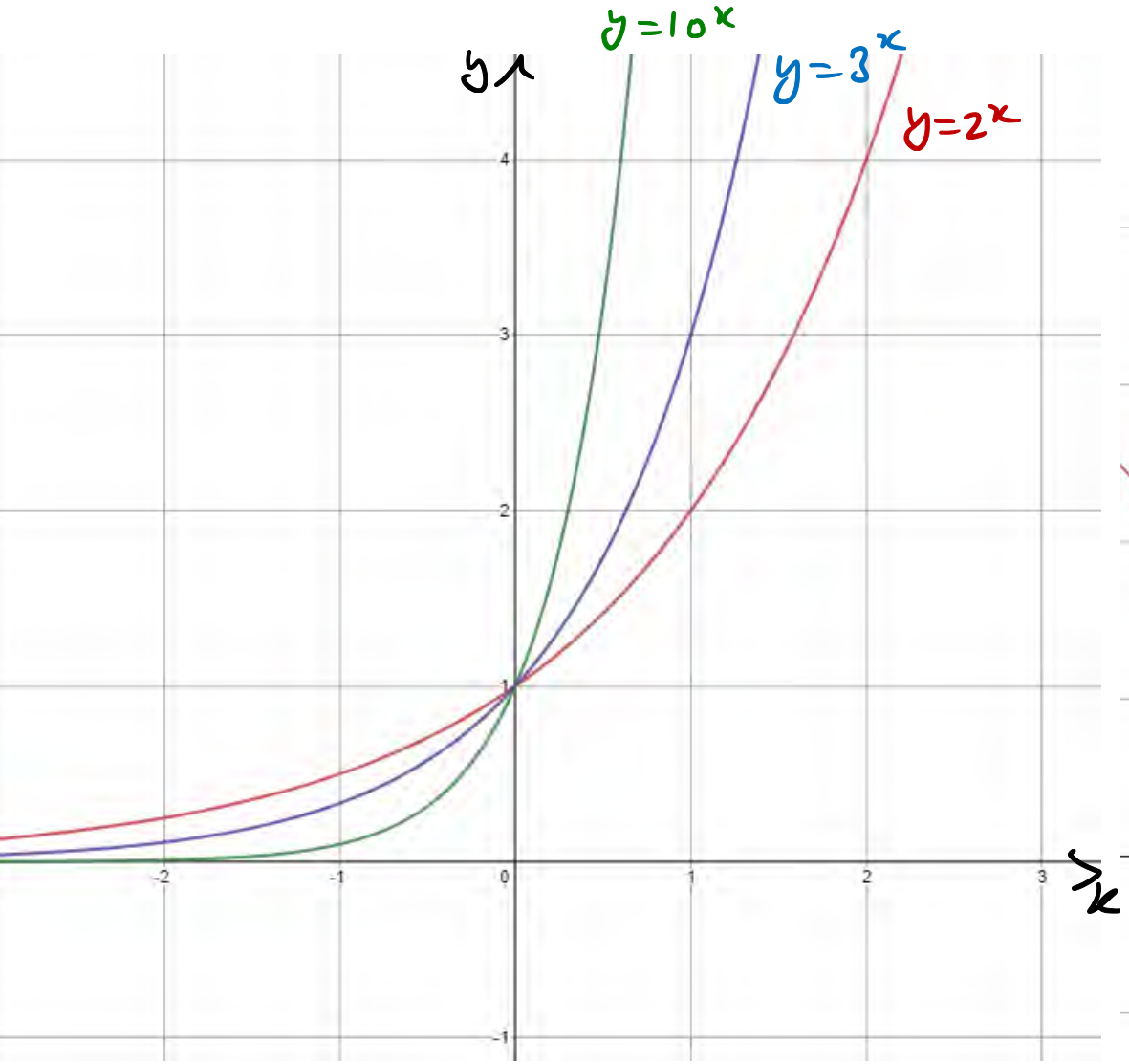
so (1) becomes $5^{5x-8-x^2} = 5^{-2}$ so $5x-8-x^2 = -2$

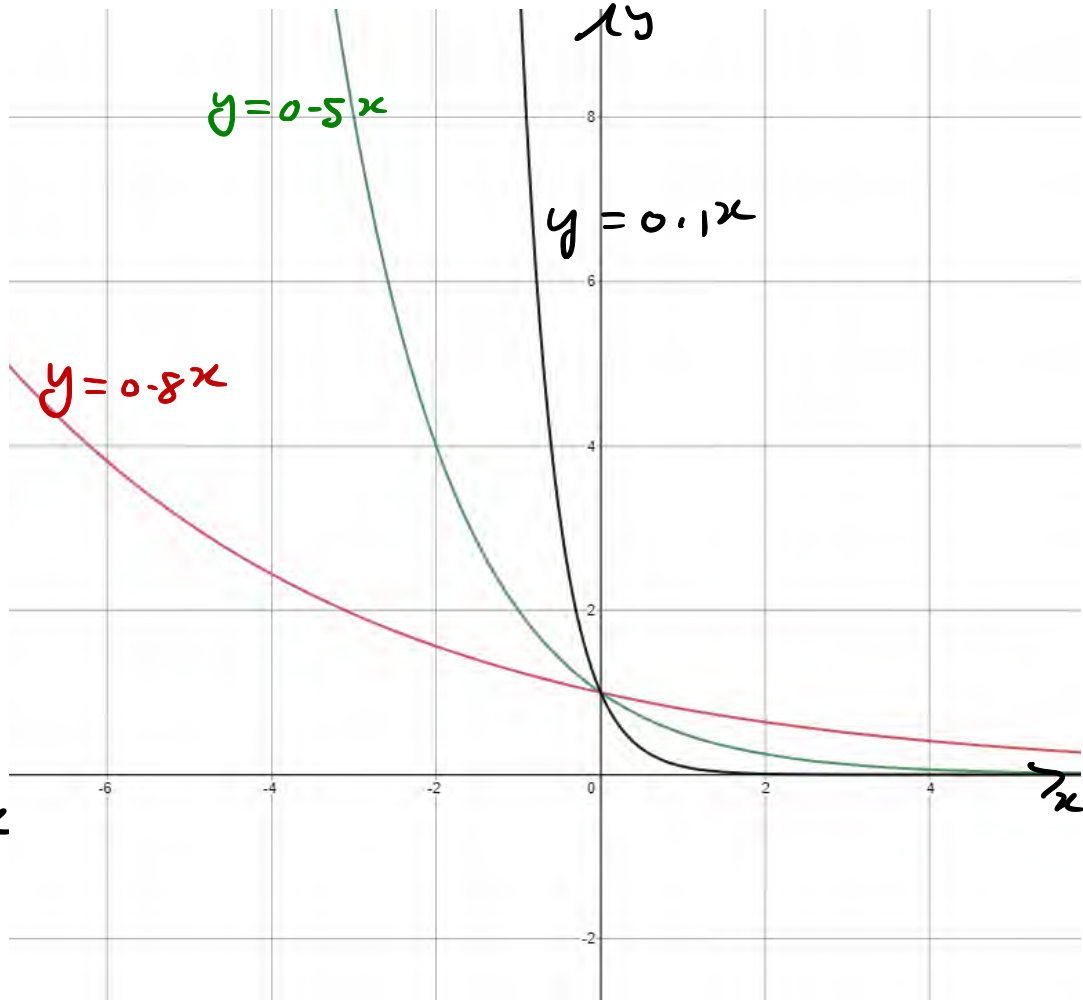
ie $x^2-5x+6=0$

$(x-2)(x-3)=0$

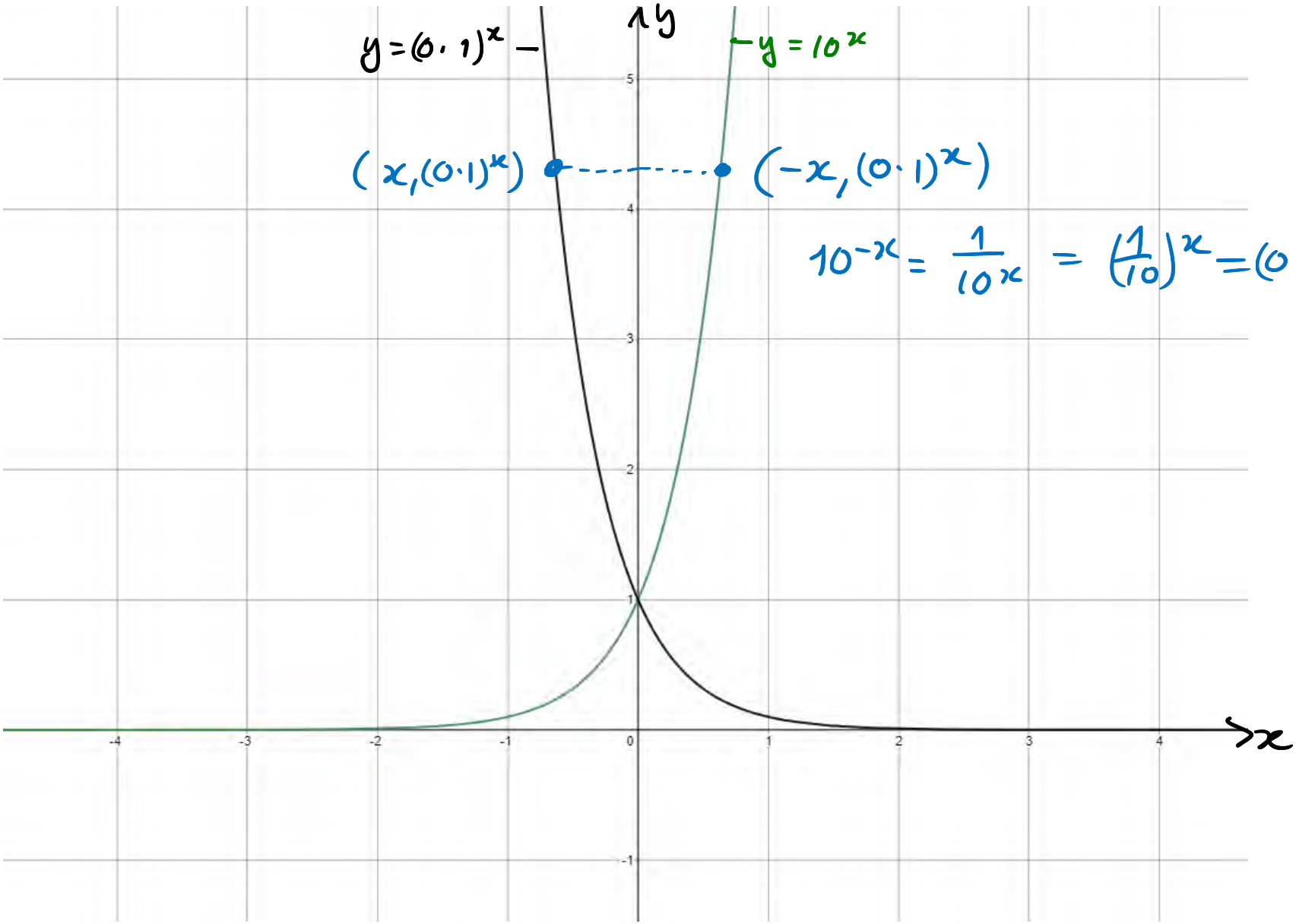
solutions $x=2$, $x=3$

Exponential Functions





Exponential Functions



$$10^{-x} = \frac{1}{10^x} = \left(\frac{1}{10}\right)^x = (0.1)^x$$